Further Pure 1 - January 2006

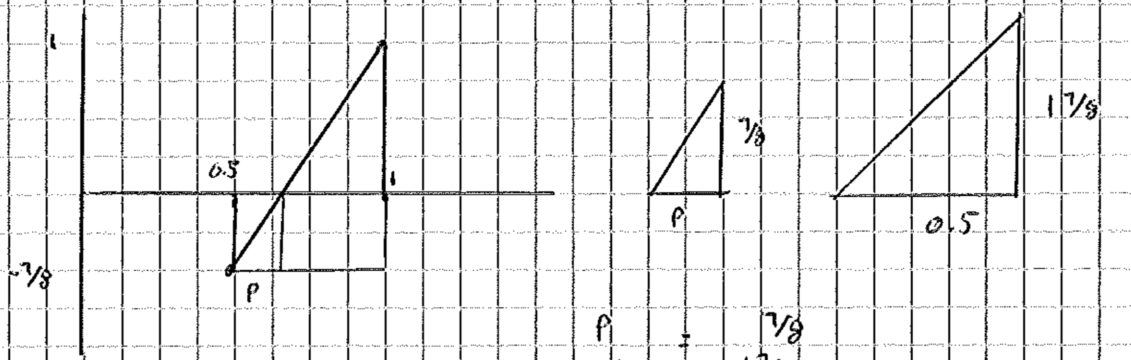
① a) Let $f(x) = x^3 + 2x - 2$

$f(0.5) = 0.5^3 + 2(0.5) - 2 = -7/8$

$f(1) = 1^3 + 2(1) - 2 = 1$

Sign change, $\therefore$ root lies between 0.5 and 1

b)



$$\frac{p}{0.5} = \frac{7/8}{1\,7/8}$$

$$\Rightarrow p = \frac{0.5 \times 7/8}{1\,7/8} = 7/30$$

$\therefore$ root approximation $= 0.5 + p = 0.5 + 7/30 = 11/15$ or $0.7\dot{3}$

② a) $\displaystyle\int_0^9 1/\sqrt{x} = \int_n^9 x^{-1/2} = \left[2x^{1/2}\right]_n^9$

$= 2\sqrt{9} - 2\sqrt{n}$

$= 6 - 2\sqrt{n}$

As $n \to 0$, $2\sqrt{n} \to 0$, so $\int \to 6$

b) $\displaystyle\int_0^9 1/x\sqrt{x} = \int_n^9 x^{-3/2} = \left[-2x^{-1/2}\right]_n^9$

$$= -\frac{2}{\sqrt{9}} - \frac{-2}{\sqrt{n}}$$

$$= \frac{-2}{3} + \frac{2}{\sqrt{n}}$$

as $n \to 0$, $2/\sqrt{n}$ does not converge, $\therefore \int$ has no value

(3) General solution for sine: $\theta = 360n + \alpha$, $\theta = 360n + (180 - \alpha)$

Key angle: $\sin^{-1}(\sin(50)) = 50°$

General solution:

$4x + 10 = 360n + 50$
$\Rightarrow 4x = 360n + 40$
$\Rightarrow x = 90n + 10$

$4x + 10 = 360n + (180 - 50)$
$\Rightarrow 4x + 10 = 360n + 130$
$\Rightarrow 4x = 360n + 120$
$\Rightarrow x = 90n + 30$

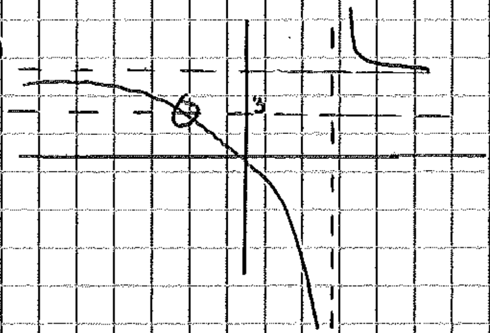
(4) a) As $x \to \infty$, $y \to 6/1$ $\therefore$ $y = 6$ is asymptote

If $x = 1$, denominator $= 0$ $\therefore$ $x = 1$ is asymptote

b)

when $x = 0$, $y = 0$

c)



Find intersection:

$\frac{6x}{x-1} = 3$

$\Rightarrow 6x = 3x - 3$

~~$\Rightarrow 9 = 3x \Rightarrow x =$~~

$\Rightarrow 3x = -3 \quad \Rightarrow \quad x = -1$

From diagram, curve < 3 when: $-1 < x < 1$

(5) a) i) $(2 + i\sqrt{5})(\sqrt{5} - i) = 2\sqrt{5} - 2i + 5i + \sqrt{5}$ $\quad -i^2\sqrt{5} = \sqrt{5}$

$= 3\sqrt{5} + 3i$

ii) Let $z = \sqrt{5} - i$ and $z^* = \sqrt{5} + i$

$(2 + i\sqrt{5})(\sqrt{5} - i) = 3\sqrt{5} + 3i$

$= 3(\sqrt{5} + i) = 3z^*$

$\therefore \sqrt{5} - i$ is a root

b) i) $\sqrt{5}+i$ (complex conjugate)

ii) $\boxed{\text{SUM}}$ $\sqrt{5}-i + \sqrt{5}+i = 2\sqrt{5}$

$\boxed{\text{PRODUCT}}$ $(\sqrt{5}-i)(\sqrt{5}+i) = 5 - -1 = 6$

iii) $x^2 - \boxed{\text{SUM}}x + \boxed{\text{PRODUCT}} = 0$

$\rightarrow p = -2\sqrt{5}, \quad q = 6$

(6) a) see sheet

b) $y = kx^n$

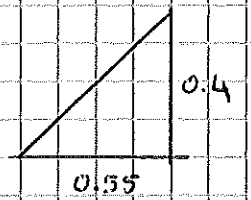
$\log_{10}(y) = \log_{10}(k) + n\log_{10}(x)$

$\rightarrow Y = a\,X + b$

$\uparrow$ n $\qquad$ $\uparrow$ $\log_{10}(k)$

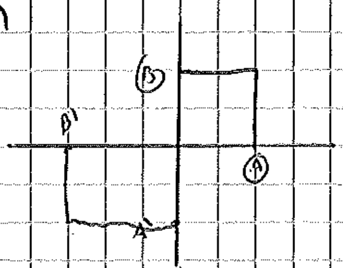
c) see sheet

d) n = gradient



$= 0.4/0.55 = 0.72$

(7) a) i)



Reflection in $y=-x$

ii) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

iii) A reflection in $y=-x$ followed by another reflection in $y=-x$ returns the object to the starting position.

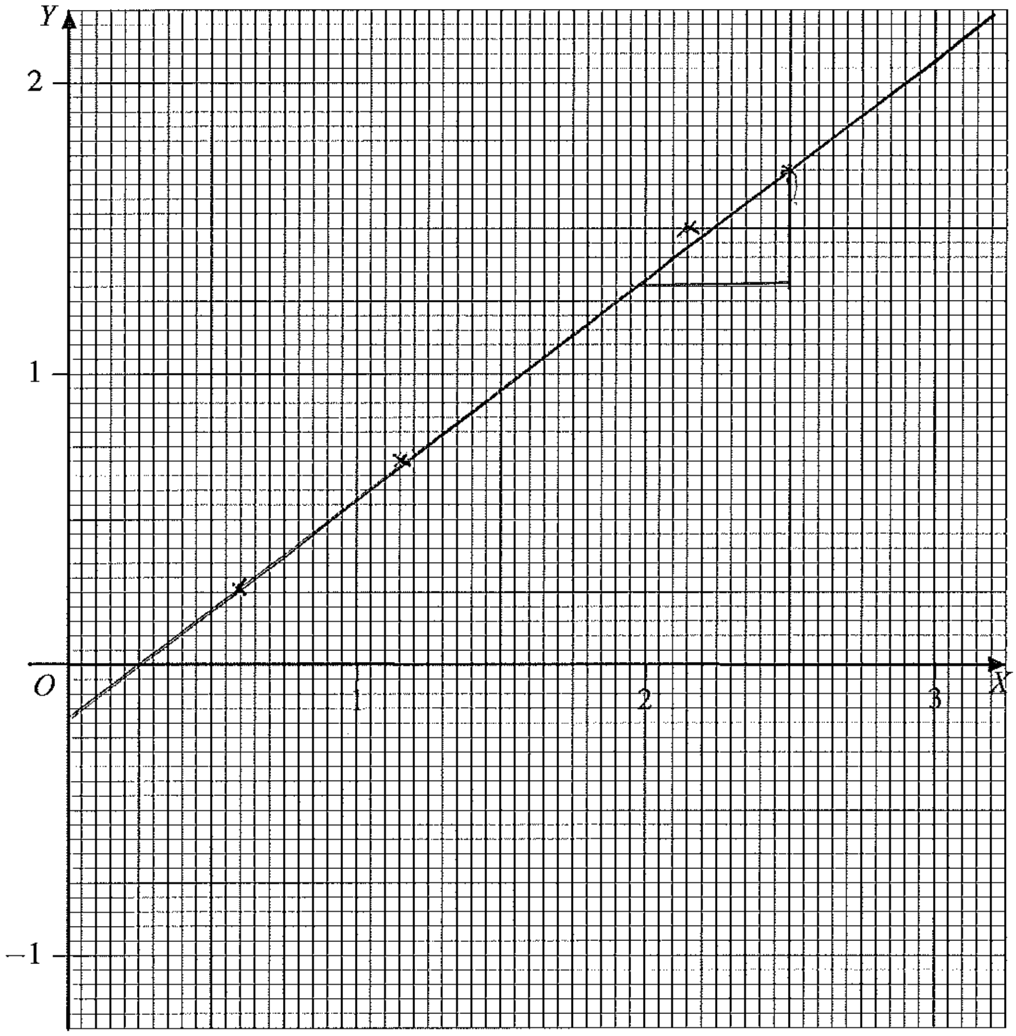
b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Figure 1 (for use in Question 6)

X	0.60	1.23	2.18	2.48
Y	0.26	0.70	1.48	1.70

Figure 2 (for use in Question 6)



$B^2 - A^2 = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\0 & 0\end{bmatrix}$

ii) $(B + A) = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} + \begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\-1 & 1\end{bmatrix}$

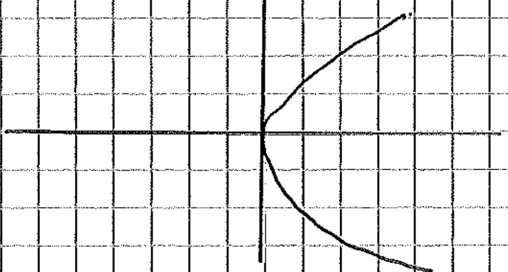
$(B - A) = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} - \begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix} = \begin{bmatrix}1 & 2\\1 & 1\end{bmatrix}$

$(B+A)(B-A)$

$\begin{bmatrix}1 & 2\\1 & 1\end{bmatrix}$

$\begin{bmatrix}1 & 0\\-1 & 1\end{bmatrix} \begin{bmatrix}1 & 2\\0 & -1\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & -1\end{bmatrix}$

⑧ a) $y^2 = 12x$



b) i) $(y-2)^2 = 12x$

ii) x and y swap

$\Rightarrow x^2 = 12y$

c) i) $y = x + c \Rightarrow (x+c)^2 = 12x$

$\Rightarrow x^2 + 2xc + c^2 = 12x$

$\Rightarrow x^2 + 2xc - 12x + c^2 = 0$

$\Rightarrow x^2 + (2c - 12)x + c^2 = 0$

ii) For tangent, $b^2 - 4ac = 0$

$\Rightarrow (2c-12)^2 - 4 \times 1 \times c^2 = 0$

$\Rightarrow 4c^2 - 48c + 144 - 4c^2 = 0$

$\Rightarrow -48c + 144 = 0$

$\Rightarrow c = 144/48 = 3$

iii) $x^2 + (2c - 12)x + c^2 = 0$

$c = 3$

$\rightarrow x^2 + (6 - 12)x + 3^2 = 0$

$\rightarrow x^2 - 6x + 9 = 0$

$\rightarrow (x - 3)(x - 3) = 0$

$\rightarrow x = 3$

$y = x + c \rightarrow y = 3 + 3 = 6$

$\therefore$ co-ordinates : $(3, 6)$

iv) if $c = 4$

$\rightarrow x^2 + (8 - 12)x + 4^2 = 0$

$\rightarrow x^2 - 4x + 16 = 0$

check discriminant :

$b^2 - 4ac$

$\rightarrow (-4)^2 - 4 \times 1 \times 16 = -48$

Discriminant is negative

$\therefore$ No real solutions

$\therefore$ Line does <u>NOT</u> intersect the curve